Measuring $\psi'' \to K_S^0 K_L^0$ as a test of the S- and D-wave mixing of charmonia

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Adding to the long standing "$\rho\pi$ puzzle" in ψ' and J/ψ decays, recently BEijing Spectrometer (BES) reported $\mathcal{B}(\psi' \to K_S^0 K_L^0)$ which is enhanced relative to the pQCD "12% rule" expectation from $\mathcal{B}(J/\psi \to K_S^0 K_L^0)$. If the enhancement is due to the mixing of the S- and D-wave charmonium states as in the $\rho\pi$ case, the newly measured $\mathcal{B}(\psi' \to K_S^0 K_L^0)$ gives a constraint on $\mathcal{B}(\psi'' \to K_S^0 K_L^0)$. It serves as a good test for the scenario of the S- and D-wave mixing in the ψ' and ψ''.

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I. INTRODUCTION

From the perturbative QCD (pQCD), it is expected that both J/ψ and $\psi(3686)$ (shortened as ψ') decaying into light hadrons are dominated by the annihilation of $c\bar{c}$ into three gluons, with widths proportional to the square of the wave function at the origin [1]. This yields the pQCD "12% rule", that is

$$Q_h = \frac{\mathcal{B}_{\psi' \to h}}{\mathcal{B}_{J/\psi \to h}} = \frac{\mathcal{B}_{\psi' \to e^+e^-}}{\mathcal{B}_{J/\psi \to e^+e^-}} \approx 12\% \quad . \tag{1}$$

The violation of the above rule was first observed in $\rho\pi$ and $K^{*+}K^- + c.c.$ modes by Mark II [2], since then BES has measured many two-body decay modes of ψ', among which some obey the 12% rule while others violate it [3]. There have been many theoretical efforts trying to solve the puzzle [4], however, none explains all the existing experimental data satisfactorily and naturally [5].

A most recent explanation of the "$\rho\pi$ puzzle" using the S- and D-wave charmonia mixing was proposed by Rosner [6]. In this scheme, the mixing of $\psi(2^3S_1)$ state and $\psi(1^3D_1)$ is in such a way which leads to almost complete cancellation of the decay amplitude of $\psi' \to \rho\pi$, and the missing $\rho\pi$ decay mode of ψ' shows up instead as enhanced decay mode of $\psi(3770)$ (shortened as ψ''). A study on the measurement of $\psi'' \to \rho\pi$ in e^+e^- experiments shows that with the decay rate predicted by the S- and D-wave mixing, the destructive interference between the three-gluon decay amplitude of the ψ'' resonance and the continuum one-photon amplitude leads to a very small cross section [7], which is in agreement with the unpublished upper limit of the $\rho\pi$ cross section at the ψ'' peak by Mark III [8]. Although this needs to be further tested by high luminosity experiment operating at the ψ'' mass energy, such as CLEO-c, it already showed that $\mathcal{B}(\psi'' \to \rho\pi)$ is most probably at the order of 10^{-4}, in agreement with the prediction of the S- and D-wave mixing scheme.

If the S- and D-wave mixing is the key for solving the $\rho\pi$ puzzle, it applies to other decay modes as well, such as pseudoscalar pseudoscalar (PP) mode like $K_S^0 K_L^0$. Recently, BES collaboration reported the $K_S^0 K_L^0$ branching ratios of J/ψ and ψ' decays [9, 10]:

$$\begin{aligned} \mathcal{B}(J/\psi \to K_S^0 K_L^0) &= (1.82 \pm 0.04 \pm 0.13) \times 10^{-4} \quad , \\ \mathcal{B}(\psi' \to K_S^0 K_L^0) &= (5.24 \pm 0.47 \pm 0.48) \times 10^{-5} \quad . \end{aligned} \tag{2}$$

These results yield $Q_{K_S^0 K_L^0} = (28.8 \pm 3.7)\%$, which is enhanced relative to the 12% rule by more than 4σ. In this paper, the $\psi' \to K_S^0 K_L^0$ enhancement is explained in the S- and D-wave charmonia mixing scheme, and the $\psi'' \to K_S^0 K_L^0$ decay rate is estimated with the inputs $\mathcal{B}(J/\psi \to K_S^0 K_L^0)$ and $\mathcal{B}(\psi' \to K_S^0 K_L^0)$. In following sections, the mixing scheme is introduced briefly, then the branching ratio of $\psi'' \to K_S^0 K_L^0$ is calculated with the measured e^+e^- and $K_S^0 K_L^0$ decay rates of J/ψ and ψ', assuming the mixing of S- and D-wave. Finally the experiment search for $\psi'' \to K_S^0 K_L^0$ is proposed.

II. S- AND D-WAVE MIXING SCHEME

To explain the measured Γ_{ee} of ψ'', it is suggested [11–13] that the mass eigenstates ψ' and ψ'' are the mixtures of the S- and D-wave of charmonia, namely $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states. In this scheme,

$$\begin{aligned} |\psi'\rangle &= |2^3S_1\rangle \cos\theta - |1^3D_1\rangle \sin\theta \; , \\ |\psi''\rangle &= |2^3S_1\rangle \sin\theta + |1^3D_1\rangle \cos\theta \; , \end{aligned} \tag{3}$$

where θ is the mixing angle between pure $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states and is fitted from the leptonic widths of ψ'' and ψ' to be either $(-27 \pm 2)^\circ$ or $(12 \pm 2)^\circ$ [6]. The latter value of θ is consistent with the coupled channel estimates [11, 14] and with the ratio of ψ' and ψ'' partial widths to $J/\psi\pi^+\pi^-$ [12]. Hereafter, the discussions in this paper are solely for the mixing angle $\theta = 12^\circ$.

As in the discussion of Ref. [6], since both hadronic and leptonic decay rates are proportional to the square of the wave function at the origin $|\Psi(0)|^2$, it is expected that if ψ' is a pure $\psi(2^3S_1)$ state, then for any hadronic final states f,

$$\Gamma(\psi' \to f) = \Gamma(J/\psi \to f)\frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \quad . \tag{4}$$

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The electronic partial width of J/ψ is expressed in potential model by [15]

$$\Gamma(J/\psi \to e^+e^-) = \frac{4\alpha^2 e_c^2}{M_{J/\psi}^2} |R_{1S}(0)|^2 \,, \tag{5}$$

with α the QED fine structure constant, $e_c = 2/3$, $M_{J/\psi}$ the J/ψ mass and $R_{1S}(0)$ the radial 1^3S_1 wave function at the origin.

ψ' is not a pure $\psi(2^3S_1)$ state, its electronic partial width is expressed as [6]

$$\begin{aligned}\Gamma(\psi' \to e^+e^-) &= \frac{4\alpha^2 e_c^2}{M_{\psi'}^2} \qquad\qquad (6)\\ &\times \left|\cos\theta R_{2S}(0) - \frac{5}{2\sqrt{2}m_c^2}\sin\theta R''_{1D}(0)\right|^2 ,\end{aligned}$$

with $M_{\psi'}$ the ψ' mass, $R_{2S}(0)$ the radial 2^3S_1 wave function at the origin and $R''_{1D}(0)$ the second derivative of the radial 1^3D_1 wave function at the origin.

If Eq. (4) holds for a pure 2^3S_1 state, $\psi'' \to f$, $\psi' \to f$ and $J/\psi \to f$ partial widths are to be

$$\begin{aligned}\Gamma(\psi'' \to f) &= \frac{C_f}{M_{\psi''}^2} |\sin\theta R_{2S}(0) + \eta\cos\theta|^2 ,\\ \Gamma(\psi' \to f) &= \frac{C_f}{M_{\psi'}^2} |\cos\theta R_{2S}(0) - \eta\sin\theta|^2 ,\\ \Gamma(J/\psi \to f) &= \frac{C_f}{M_{J/\psi}^2} |R_{1S}(0)|^2 \ , \qquad (7)\end{aligned}$$

where C_f is a common factor for the final state f, $M_{\psi''}$ the ψ'' mass, and $\eta = |\eta|e^{i\phi}$ is a complex parameter with ϕ being the relative phase between $\langle f|1^3D_1\rangle$ and $\langle f|2^3S_1\rangle$.

III. UPPER AND LOWER BOUNDS OF $\mathcal{B}(\psi'' \to K_S^0 K_L^0)$

With Eqs. (5, 6, 7), the following two equations are derived :

$$\begin{aligned}\frac{\Gamma(\psi' \to f)}{\Gamma(J/\psi \to f)} &= \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(J/\psi \to e^+e^-)} \qquad\qquad (8)\\ &\times \left| \frac{\cos\theta R_{2S}(0) - \eta\sin\theta}{\cos\theta R_{2S}(0) - \frac{5}{2\sqrt{2}m_c^2}\sin\theta R''_{1D}(0)} \right|^2 ,\end{aligned}$$

and

$$\frac{\Gamma(\psi'' \to f)}{\Gamma(\psi' \to f)} = \frac{M_{\psi'}^2}{M_{\psi''}^2} \left|\frac{\sin\theta R_{2S}(0) + \eta\cos\theta}{\cos\theta R_{2S}(0) - \eta\sin\theta}\right|^2 . \tag{9}$$

It is easy to see that if $\theta = 0$, i.e. ψ' were a pure $\psi(2^3S_1)$ state, Eq. (8) becomes Eq. (4).

In the following, the discussion focuses on $f = K_S^0 K_L^0$ final state. The partial widths of ψ' and J/ψ to e^+e^- [16] and $K_S^0 K_L^0$ [9, 10] are all measured by experiments; $R_{2S}(0) = 0.734$ GeV$^{3/2}$ and $5R''_{1D}(0)/(2\sqrt{2}m_c^2) = 0.095$ GeV$^{3/2}$ are given in Ref. [6], so for the final state $K_S^0 K_L^0$, Eq. (8) has only one unknown variable η. Since η is complex, for any given phase, its module can be determined. Then with η substituting into Eq. (9), $\Gamma(\psi'' \to K_S^0 K_L^0)$ can be calculated.

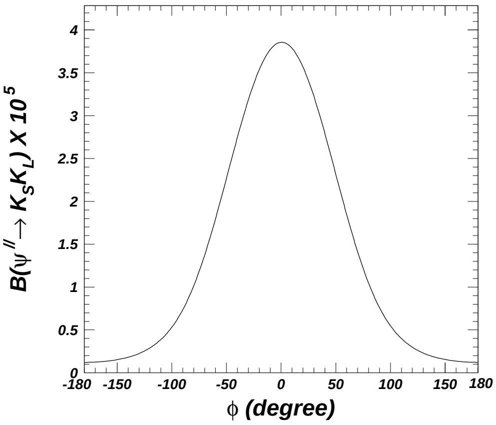

FIG. 1: The variation of $\mathcal{B}(\psi'' \to K_S^0 K_L^0) \times 10^5$ against the phase ϕ (in degree).

Since the phase of η is a free parameter, so the decay rate of $\psi'' \to K_S^0 K_L^0$ is constrained in a range. According to Eqs. (8) and (9), the variation of branching ratio against the phase is shown in Fig. 1, from which we see that

$$0.12 \pm 0.07 \leq 10^5 \times \mathcal{B}(\psi'' \to K_S^0 K_L^0) \leq 3.8 \pm 1.1 \ . \tag{10}$$

Here the upper bound corresponds to $\phi = 0°$ and the lower bound to $\phi = \pm 180°$. The uncertainties are due to the mixing angle θ, the measurements of $\mathcal{B}(\psi' \to K_S^0 K_L^0)$ and $\mathcal{B}(J/\psi \to K_S^0 K_L^0)$, with the first two dominate.

IV. EXPERIMENTAL TEST

It it instructive to look at the range of the phase ϕ from other decay modes, such as $\rho\pi$. The recent phenomenological estimation [17] gives the branching ratio of $\psi' \to \rho\pi$ at the level of 10^{-4}, which indicates the almost complete cancellation between $\cos\theta R_{2S}(0)$ and $\eta\sin\theta$ in Eq. (7). In another word, the small $\mathcal{B}(\psi' \to \rho\pi)$ means the phase ϕ of η is around zero. With incomplete cancellation between $\cos\theta R_{2S}(0)$ and $\eta\sin\theta$ which results in $\mathcal{B}(\psi' \to \rho\pi) = 1.11 \times 10^{-4}$ [17], and latest results by BES of $\mathcal{B}(J/\psi \to \rho\pi) \sim 2.1\%$ [18], ϕ is constrained to be less than $11°$. As a pedagogical guess, ϕ is expected to be small for other decay modes too. In such case, the prediction $\mathcal{B}(\psi'' \to K_S^0 K_L^0)$ would be close to the upper bound in Eq. (10), that is

$$\mathcal{B}(\psi'' \to K_S^0 K_L^0) \approx (3.8 \pm 1.1) \times 10^{-5} \ . \tag{11}$$

Currently, BES has accumulated about 20 pb^{-1} data while CLEO-c has collected 55 pb^{-1} data at ψ'' peak. By virtue of Eq. (11), assuming 40% efficiency for detecting $K^0_S \to \pi^+\pi^-$, then one expects 1.7 events from BES and 4.6 events from CLEO-c. Utilizing these samples, most probably an upper limit can be set by BES, while the signal can be seen at CLEO-c. With the expected larger ψ'' data sample of several fb^{-1} [19] in immediate future, CLEO-c can give a definite answer for prediction of Eq. (11), or test the lower bound of Eq. (10) in case the phase ϕ is not small.

V. DISCUSSION

In the S- and D-wave mixing scheme, the observed $\psi' \to K^0_S K^0_L$ enhancement relative to the 12 % rule implies a $\psi'' \to K^0_S K^0_L$ decay branching ratio at the order of 10^{-6} to 10^{-5}. So the measurement of $\mathcal{B}(\psi'' \to K^0_S K^0_L)$ will provide a clear-cut test of the S- and D-wave mixing scenario.

Unlike the $\rho\pi$ modes, $K^0_S K^0_L$ mode of the 1^{--} charmonium decay is only through strong interaction due to SU(3) symmetry [20]. There is no complication of electromagnetic interaction and continuum one-photon annihilation as well as the interference between them [21]. So the observed $K^0_S K^0_L$ in e^+e^- experiment is completely from resonance decays.

If the ψ' and ψ'' are indeed the S- and D-wave charmonia mixtures, not only the vector pseudoscalar [6] and the pseudoscalar pseudoscalar modes will be affected, but all the other modes in ψ' decays will be affected as well, such as vector tensor, axial-vector pseudoscalar and so forth. For the decay modes which have been measured both at ψ' and J/ψ, the corresponding branching ratio at ψ'' can be evaluated under the assumption of pQCD. Then the measurements at ψ'' provide a test for the mixing scheme, at the same time help to reveal the charmonium decay dynamics and the relation between J/ψ and ψ' decays.

The mixing scheme is a simple and natural model, it will provide a new angle of purview of understanding the $\rho\pi$ puzzle between J/ψ and ψ' decays, and the non-$D\overline{D}$ decay of ψ''.

VI. SUMMARY

In this paper, the S- and D-wave mixing scheme of charmonium states is applied on $\psi' \to K^0_S K^0_L$ to explain its enhancement relative to the pQCD 12% rule, and the branching ratio of $\psi'' \to K^0_S K^0_L$ is predicted. It is suggested that with the data samples collected currently and the larger data sample expected from CLEO-c soon, the mixing scheme is to be tested.

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